

# Distance, entropy and similarity measures of Type-2 soft sets

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## Abstract

The concept of Type-2 soft sets had been proposed as a generalization of Molodstov's soft sets. In this paper some shortcomings of some existing distance measures for Type-1 soft sets have been shown and accordingly some new distance measures have been proposed. The axiomatic definitions for distance, entropy and similarity measures for Type-2 soft sets have been introduced and a couple of such measures have been defined. Also the applicability of one of the proposed similarity measures have been demonstrated by showing its utility as an effective tool in a decision making problem.

Keywords: Soft sets, Type-2 soft sets, distance, entropy, similarity.

## 1 Introduction

Soft set theory, introduced by Molodstov [12], emerged as a revolutionary technique in dealing with intrinsic imprecision with the help of adequate parameterization. Thereby the theory of soft sets not only catered to the shortcoming encountered by fuzzy sets [14] possibly due to the lack of a parameterization tool but in his work, Molodstov had also shown that fuzzy sets were special types of soft sets, the parameter set being considered over the unit interval  $[0, 1]$ . Later, Maji et. al [7] presented a detailed theoretical study on soft sets. Thereafter the theory of soft sets has undergone rapid developments in different directions ([1]-[10],[13]).

The organization of the paper is as follows:

Section 1 is the introductory portion, Section 2 is dedicated to recalling some useful preliminary results. The notions of distance, entropy and similarity measures for type-2 soft sets are defined in Sections 3, 4 and 5 respectively. In Section 6, an application of the proposed similarity measure in a decision making problem is shown. Section 7 concludes the paper.

## 2 Useful preliminaries

Before introducing the proposed measures we first brush up some preliminary results that would be useful for future purposes.

**Definition 2.1.** [7] Let  $X$  be an initial universe and  $E$  be a set of parameters. Let  $\mathcal{P}(X)$  denote the power set of  $X$  and  $A \subset E$ . A pair  $(F, A)$  is called a soft set if  $F$  is a mapping of  $A$  into  $\mathcal{P}(X)$ .

**Definition 2.2.** [4] The bi-intersection of two T1SS  $(F, A)$  and  $(G, B)$  is defined as  $(F, A) \tilde{\cap} (G, B) = (H, C)$ , where  $C = A \cap B$  such that for all  $\alpha \in C$ ,  $H(\alpha) = F(\alpha) \cap G(\alpha)$ .

**Definition 2.3.**  $(F, A) \tilde{\cap} (G, B) = \begin{cases} (H, A \cap B) & \text{if } A \cap B \neq \varphi \\ \hat{\phi}, \text{ the empty function} & \text{if } A \cap B = \varphi \end{cases}$   
where  $H(\alpha) = F(\alpha) \cap G(\alpha)$ , for all  $\alpha \in A \cap B$ .

**Definition 2.4.** [10] A T1SS  $(F, A)$  is said to be deterministic if it satisfies the conditions

- (i)  $\cup_{e \in A} F(e) = X$
- (ii)  $F(e) \cap F(f) = \varphi$ ,  $e, f \in A$ ,  $e \neq f$ .

**Definition 2.5.** [3] Let  $R$  be the set of real numbers,  $\mathcal{P}(R)$  be the collection of all non-empty bounded subsets of  $R$  and  $A$  be a set of parameters. Then a mapping  $F : A \rightarrow \mathcal{P}(R)$  is called a soft real set, which is denoted by  $(F, A)$ . In particular, if  $(F, A)$  is a singleton soft set, then identifying  $(F, A)$  with the corresponding soft element, it is called a soft real number.

**Definition 2.6.** [2] Let  $(X, E)$  be an initial soft universe and  $A, E_A \subset E$  be two sets of parameters. Suppose that  $S(X, E_A)$  denotes the set of all T1SS over the soft universe  $(X, E_A)$ . Then a type-2 soft set over  $(X, E)$  is a mapping  $\mathcal{F} : A \rightarrow S(X, E_A)$ . It is denoted by  $[\mathcal{F}, A]$ .

In this case the set of parameters  $A$  is referred to as the *primary set of parameters* while the set of parameters  $E_A$  is known as the *underlying set of parameters*. Thus, for a type-2 soft set, corresponding to each parameter  $\alpha \in A$ , there exists a TISS  $(F_\alpha, E_A^\alpha) \in S(X, E_A)$  where  $F_\alpha : E_A^\alpha \rightarrow \mathcal{P}(X)$  and  $E_A^\alpha \subset E_A$  such that  $\cup_{\alpha \in A} E_A^\alpha = E_A$ .

**Example 2.7.** Let  $X = \{h_1, h_2, h_3, h_4, h_5\}$  be a set of five houses. Suppose  $E = \{\text{beautiful, single storeyed, wooden, in good repair, spacious, near the market, in green surroundings, furnished, luxurious, with good security, with pool}\}$

Let  $A = \{\text{beautiful, luxurious}\}$  and  $E_A = \{\text{wooden, in green surroundings, with good security}\}$  be two sets of parameters such that  $A, E_A \subset E$ . A T2SS  $[\mathcal{F}, A]$  can be defined as follows:

$$\begin{aligned} \mathcal{F}(\text{beautiful}) &= \left\{ \frac{\text{wooden}}{\{h_2, h_5\}}, \frac{\text{in green surroundings}}{\{h_1, h_2, h_3, h_4\}} \right\} \\ \mathcal{F}(\text{luxurious}) &= \left\{ \frac{\text{wooden}}{\{h_5\}}, \frac{\text{with good security}}{\{h_1, h_3, h_5\}} \right\} \end{aligned}$$

Another T2SS  $[\mathcal{G}, B]$  can be defined on the same soft universe  $(X, E)$  as,

$$\begin{aligned} \mathcal{G}(\text{spacious}) &= \left\{ \frac{\text{wooden}}{\{h_5\}}, \frac{\text{with pool}}{\{h_3, h_5\}} \right\} \\ \mathcal{G}(\text{beautiful}) &= \left\{ \frac{\text{wooden}}{\{h_2, h_5\}}, \frac{\text{near the market}}{\{h_4\}}, \frac{\text{in green surroundings}}{\{h_2, h_3\}} \right\} \end{aligned}$$

In this case,  $B = \{\text{spacious, beautiful}\}$  and  
 $E_B = \{\text{wooden, with pool, near the market, in green surroundings}\}$

**Remark 2.8.[2]** Type 2 fuzzy sets introduced by L. A. Zadeh may be considered as special types of type-2 soft sets when the parameters are considered over  $[0, 1]$ . Let  $X$  be the universe under consideration. The point-valued representation of a type-2 fuzzy set, denoted by  $\tilde{A}$ , is given in [11] as,

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}$$

where  $\mu_{\tilde{A}}(x, u)$  is the type 2 membership function for all  $x \in X$ . Then, for any  $\alpha \in [0, 1]$ , the corresponding  $\alpha$ -plane of  $\tilde{A}$  is given by,

$$\tilde{A}_\alpha = \{(x, u), \mu_{\tilde{A}}(x, u) \geq \alpha | \forall x \in X, \forall u \in J_x \subseteq [0, 1]\}$$

For a particular  $\alpha \in [0, 1]$ , construct the set,

$$S_\alpha = \{u : (x, u) \in \tilde{A}_\alpha\}$$

Define a mapping  $F_\alpha : S_\alpha \rightarrow \mathcal{P}(X)$  by

$$F_\alpha(u) = \{x : (x, u) \in \tilde{A}_\alpha\}, u \in S_\alpha.$$

Thus,  $(F_\alpha, S_\alpha)$  constitutes a type-1 soft set over  $(X, [0, 1])$ .

Next, define a mapping  $\mathcal{F} : [0, 1] \rightarrow S(X, [0, 1])$  such that  $\mathcal{F}(\alpha) = (F_\alpha, S_\alpha)$ .

Thus,  $[\mathcal{F}, [0, 1]]$  is a type-2 soft set over  $(X, [0, 1])$ .

**Definition 2.9.** The various operations over T2SS are defined as,

Containment:  $[\mathcal{F}, A] \subseteq [\mathcal{G}, B]$  iff  $A \subseteq B$  and  $\mathcal{F}(\alpha) \tilde{\subseteq} \mathcal{G}(\alpha)$ , for all  $\alpha \in A$ .

Thus, when  $[\mathcal{F}, A] \subseteq [\mathcal{G}, B]$  we have,  $\mathcal{F}(\alpha) \tilde{\subseteq} \mathcal{G}(\alpha)$ , for each  $\alpha \in A$  and so it follows that  $E_A^\alpha \subseteq E_B^\alpha$  and  $F_\alpha(\beta) = G_\alpha(\beta)$ , for each  $\alpha \in A$  and  $\beta \in E_A^\alpha$ . Thus,  
 $E_A = \cup_{\alpha \in A} E_A^\alpha \subseteq \cup_{\alpha \in B} E_B^\alpha = E_B$

Equality:  $[\mathcal{F}, A] = [\mathcal{G}, B]$  iff  $[\mathcal{F}, A] \subseteq [\mathcal{G}, B]$  and  $[\mathcal{G}, B] \subseteq [\mathcal{F}, A]$ .

$$\text{Union: } [\mathcal{F}, A] \sqcup [\mathcal{G}, B] = \begin{cases} \mathcal{F}(\alpha) & \alpha \in A - B \\ \mathcal{G}(\alpha) & \alpha \in B - A \\ \mathcal{F}(\alpha) \tilde{\cup} \mathcal{G}(\alpha) & \alpha \in A \cap B \end{cases}$$

$[\mathcal{F}, A] \sqcup [\mathcal{G}, B]$  is a T2SS with  $A \cup B$  being the set of primary parameters and the set of underlying parameters being  $\cup_{\alpha \in A \cup B} (E_A^\alpha \cup E_B^\alpha) = E_{A \cup B}$ .

Intersection:

$$[\mathcal{F}, A] \cap [\mathcal{G}, B] = \begin{cases} \mathcal{F}(\alpha) \tilde{\cap} \mathcal{G}(\alpha), \alpha \in A \cap B & \text{whenever } A \cap B \neq \varnothing \\ \hat{\phi}, \text{ the empty function} & \text{if } A \cap B = \varnothing, \text{ the null set} \end{cases}$$

Whenever  $A \cap B \neq \varnothing$ , the intersection between two T2SS is defined below:

**Definition 2.10.** A T2SS  $[\mathcal{F}, A]$ , is said to be an absolute T2SS if for each  $\alpha \in A$ ,  $\mathcal{F}(\alpha)$  is an absolute T1SS. It is denoted by  $\hat{A}$ .

**Definition 2.11.** A T2SS  $[\mathcal{F}, A]$ , is said to be a null T2SS if for each  $\alpha \in A$ ,  $\mathcal{F}(\alpha)$  is a null T1SS. It is denoted by  $\tilde{\Phi}$ .

**Definition 2.12.** Two T2SS  $[\mathcal{F}, A]$  and  $[\mathcal{G}, B]$  are said to be *disjoint* if  $A \cap B = \varnothing$ . They are said to be *weakly disjoint* if  $A \cap B \neq \varnothing$  but corresponding to each  $\alpha \in A \cap B$ ,  $E_A^\alpha \cap E_B^\alpha = \varnothing$  i.e. in such cases  $[\mathcal{F}, A] \cap [\mathcal{G}, B] = \tilde{\phi}$ , the null T1SS. They are said to be *elementwise disjoint* if  $A \cap B \neq \varnothing$ ,  $E_A \cap E_B \neq \varnothing$

and for each  $\alpha \in A \cap B$ ,  $\mathcal{F}(\alpha) \tilde{\cap} \mathcal{G}(\alpha)$  is a null T1FS. i.e. corresponding to each primary parameter  $\alpha \in A \cap B$ ,  $(F_\alpha, E_A^\alpha) \tilde{\cap} (G_\alpha, E_B^\alpha) = \tilde{\Phi}$  over the set of parameters  $E_{A \cap B}$ .

### 3 Distance measures

**Definition 3.1.** [5] Let  $(F, A)$ ,  $(G, B)$  and  $(H, C)$  be any 3 soft sets in a soft space  $(X, E)$  and  $d : S(X, E) \times S(X, E) \rightarrow \mathbb{R}^+$ , a mapping. Then

- (1)  $d$  is said to be a quasi-metric if it satisfies
  - (M1)  $d((F, A), (G, B)) \geq 0$
  - (M2)  $d((F, A), (G, B)) = d((G, B), (F, A))$
- (2) A quasi-metric  $d$  is said to be a semi-metric if
  - (M3)  $d((F, A), (G, B)) + d((G, B), (H, C)) \geq d((F, A), (H, C))$
- (3) A semi-metric  $d$  is said to be a pseudo-metric if
  - (M4)  $(F, A) = (G, B) \Rightarrow d((F, A), (G, B)) = 0$
- (4) A pseudo metric  $d$  is said to be a metric if
  - (M5)  $d((F, A), (G, B)) = 0 \Rightarrow (F, A) = (G, B)$ .

Following this definition, Kharal[5] had introduced several new measures of calculating the distance between T1SS. In his work, out of these measures, Kharal had stated that only the Euclidean distance and the Normalized Euclidean distance between two T1SS were metrics (refer to *Definition 19, Proposition 20*, pages 8 – 9)[5]. These two distances were respectively defined in[5] as

Euclidean distance[5]:

$$e((F, A), (G, B)) = |A \Delta B| + \sqrt{\sum_{\alpha \in A \cap B} |F(\alpha) \Delta G(\alpha)|^2}$$

Normalized Euclidean distance[5]:

$$q((F, A), (G, B)) = \frac{|A \Delta B|}{\sqrt{|A \cup B|}} + \sqrt{\sum_{\alpha \in A \cap B} \chi(\alpha)}$$

$$\text{where } \chi(\alpha) = \begin{cases} \frac{|F(\alpha) \Delta G(\alpha)|^2}{|F(\alpha) \cup G(\alpha)|} & \text{if } F(\alpha) \cup G(\alpha) \neq \varphi \\ 0 & \text{otherwise} \end{cases}$$

However, in this respect, a deeper study reveals that these two above mentioned measures of distance contain fallacies since they do not always satisfy the triangle inequality (M3). In order to establish our point, we provide a counter-example in support of our argument as follows:

**Example 3.2.** Let  $(F, A)$ ,  $(G, B)$  and  $(H, C)$  be T1SS defined over the universe  $U = \{x_1, x_2, x_3, x_4, x_5\}$  and the set of parameters  $E = \{\alpha_1, \alpha_2, \alpha_3\}$  such that:

$$(F, A) = \{(\alpha_1, \{x_1, x_2\})\}; (G, B) = \{(\alpha_2, \{x_2, x_3\}), (\alpha_3, \{x_1, x_4\})\};$$

$$(H, C) = \{(\alpha_1, \{x_3, x_4, x_5\}), (\alpha_2, \{x_2, x_3\}), (\alpha_3, \{x_1, x_3, x_4\})\}$$

On calculation we get  $e((F, A), (G, B)) = 3$ ,  $e((G, B), (H, C)) = 2$  and  $e((F, A), (H, C)) = 7$ . i.e.  $e((F, A), (H, C)) > e((F, A), (G, B)) + e((G, B), (H, C))$ . Also,  $q((F, A), (G, B)) = 1.155$ ,  $q((G, B), (H, C)) = 1.155$  and  $q((F, A), (H, C)) = 3.391$  i.e.  $q((F, A), (H, C)) > q((F, A), (G, B)) + q((G, B), (H, C))$ .

Thus, the Euclidean and the Normalized Euclidean distances, which were referred to as metrics are, in reality, not metrics since they violate the triangle inequality.

In view of the above situation, we hereby propose some distance measures between two T1SS as

**Definition 3.3.** The parameter based distance measure is defined as  $d_p((F, A), (G, B)) = |A \cup B| - |A \cap B| + |F^\# \cup G^\#| - |F^\# \cap G^\#|$  where  $F^\# = \cup_{\alpha \in A} \{F(\alpha)\}$  and  $G^\# = \cup_{\beta \in B} \{G(\beta)\}$

**Theorem 3.4.**  $d_p((F, A), (G, B))$  is a metric.

Proof: We only give the proof of the Triangle inequality since the other proofs are straight-forward.

Consider any three T1SS  $(F, A), (G, B), (H, C) \in S(X, E)$ . Now,

$$\begin{aligned} & |A \cup B| - |A \cap B| + |B \cup C| - |B \cap C| - |A \cup C| + |A \cap C| \\ &= 2(|B| - |A \cap B| - |B \cap C| + |A \cap C|) \\ &\text{Also, } B = (B \cap (A \cup C)) \cup (B \cap (A \cup C)^c) \\ &\Rightarrow |B| = |B \cap (A \cup C)| + |B \cap (A \cup C)^c| = |(A \cap B) \cup (B \cap C)| + |B \cap (A^c \cap C^c)| = \\ &= |A \cap B| + |B \cap C| - |A \cap B \cap C| + |B \cap A^c \cap C^c| \\ &\text{i.e. } |B| - |A \cap B| - |B \cap C| = -|A \cap B \cap C| + |B \cap A^c \cap C^c| \\ &\text{So, } 2(|B| - |A \cap B| - |B \cap C| + |A \cap C|) = 2(|A \cap C| - |A \cap B \cap C| + |B \cap A^c \cap C^c|) \geq 0, \\ &\text{since } (A \cap B \cap C) \subseteq A \cap C \end{aligned}$$

Hence it follows that

$$|A \cup C| - |A \cap C| \leq |A \cup B| - |A \cap B| + |B \cup C| - |B \cap C|$$

Similarly, it can be proved that,

$$|F^\# \cup H^\#| - |F^\# \cap H^\#| \leq |F^\# \cup G^\#| - |F^\# \cap G^\#| + |G^\# \cup H^\#| - |G^\# \cap H^\#|$$

**Definition 3.5.** The matrix-representation based distance measure is defined as

$$\begin{aligned} d_m((F, A), (G, B)) &= |A \cup B| - |A \cap B| + \sum_{\alpha \in A \cup B} \sum_{x \in X} |F(\alpha)(x) - G(\alpha)(x)|, \\ \text{where } F(\alpha)(x) &= \begin{cases} 1 & \text{if } x \in F(\alpha) \\ 0 & \text{otherwise} \end{cases}; G(\alpha)(x) = \begin{cases} 1 & \text{if } x \in G(\alpha) \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

**Theorem 3.6.** The measure  $d_m$  is a metric.

Proof: We only give an outline of the proof of the triangle inequality since the rest of the proofs are straight-forward:

Consider any three T1SS  $(F, A), (G, B), (H, C) \in S(X, E)$ . Proceeding in an exactly same way as the proof of Theorem 3.4. , we can show that for any three parameter sets  $A, B, C$

$$|A \cup C| - |A \cap C| \leq |A \cup B| - |A \cap B| + |B \cup C| - |B \cap C|$$

$$\begin{aligned} & \text{Also, } |F(\alpha)(x_i) - H(\alpha)(x_i)| = |F(\alpha)(x_i) - G(\alpha)(x_i) + G(\alpha)(x_i) - H(\alpha)(x_i)| \leq \\ & |F(\alpha)(x_i) - G(\alpha)(x_i)| + |G(\alpha)(x_i) - H(\alpha)(x_i)| \\ & \Rightarrow d_m((F, A), (H, C)) \leq d_m((F, A), (G, B)) + d_m((G, B), (H, C)) \end{aligned}$$

We now proceed to define the distance measure between two T2SS as a generalization of the distance measures for T1SS.

**Definition 3.7.** A mapping  $D : S_2(X, E) \times S_2(X, E) \rightarrow \mathbb{R}^+$ , where  $S_2(X, E)$  denotes the set of all type-2 soft sets over the soft universe  $(X, E)$ , is said to be a distance measure between any two T2SS if and only if for all  $[\mathcal{F}, A], [\mathcal{G}, B], [\mathcal{H}, C] \in S_2(X, E)$  it satisfies the following conditions:

- (d1)  $D([\mathcal{F}, A], [\mathcal{G}, B]) = D([\mathcal{G}, B], [\mathcal{F}, A])$
- (d2)  $D([\mathcal{F}, A], [\mathcal{G}, B]) \geq 0$
- (d3)  $D([\mathcal{F}, A], [\mathcal{G}, B]) = 0$  iff  $[\mathcal{F}, A] = [\mathcal{G}, B]$ .
- (d4)  $D([\mathcal{F}, A], [\mathcal{H}, C]) \leq D([\mathcal{F}, A], [\mathcal{G}, B]) + D([\mathcal{G}, B], [\mathcal{H}, C])$

In addition to the above conditions if a distance measure satisfies the following property viz.

- (d5)  $D([\mathcal{F}, A], [\mathcal{G}, B]) \leq 1$

it is said to be a Normalized distance measure.

**Theorem 3.8.** The parameter based distance measure for T2SS defined as  $D_p([\mathcal{F}, A], [\mathcal{G}, B]) = |A \cup B| - |A \cap B| + |E_A \cup E_B| - |E_A \cap E_B| + |F^{##} \cup G^{##}| - |F^{##} \cap G^{##}|$  where  $F^{##} = \cup_{\alpha \in A} \cup_{\beta \in E_A^\alpha} \{F_\alpha(\beta)\}$  and  $G^{##} = \cup_{\alpha \in B} \cup_{\beta \in E_B^\alpha} \{G_\alpha(\beta)\}$  is a metric.

Proofs are similar to those of Theorem 3.4.

**Theorem 3.9.** The mapping  $D_m : S_2(X, E) \times S_2(X, E) \rightarrow \mathbb{R}^+$  defined by

$$D_m([\mathcal{F}, A], [\mathcal{G}, B]) = |A \cup B| - |A \cap B| + |E_A \cup E_B| - |E_A \cap E_B| + \sum_{\alpha \in A \cap B} \sum_{\beta \in E_A^\alpha \cap E_B^\alpha} \sum_{x \in X} |F_\alpha(\beta)(x) - G_\alpha(\beta)(x)|$$

$$\text{where } F_\alpha(\beta)(x) = \begin{cases} 1 & \text{if } x \in F_\alpha(\beta) \\ 0 & \text{otherwise} \end{cases}; G_\alpha(\beta)(x) = \begin{cases} 1 & \text{if } x \in G_\alpha(\beta) \\ 0 & \text{otherwise} \end{cases}$$

is a distance measure between the T2SS  $[\mathcal{F}, A]$  and  $[\mathcal{G}, B]$ . It is the matrix-representation based distance measure between T2SS.

**Theorem 3.10.** The mappings  $ND_p : S_2(X, E) \times S_2(X, E) \rightarrow [0, 1]$  and  $ND_m : S_2(X, E) \times S_2(X, E) \rightarrow [0, 1]$  defined as

$$ND_p([\mathcal{F}, A], [\mathcal{G}, B]) = \frac{1}{|X| \times |A \cup B| \times |E_A \cup E_B|} \times D_p([\mathcal{F}, A], [\mathcal{G}, B])$$

$$ND_m([\mathcal{F}, A], [\mathcal{G}, B]) = \frac{1}{|X| \times |A \cup B| \times |E_A \cup E_B|} \times D_m([\mathcal{F}, A], [\mathcal{G}, B])$$

are distance measures. Furthermore, these measures are the normalized parameter-based and normalized matrix-representation based distance measures for T2SS.

**Remark 3.11.** If, in particular, for  $[\mathcal{F}, A], [\mathcal{G}, B], [\mathcal{H}, C] \in S_2(X, E)$ ,  $[\mathcal{F}, A] \subseteq [\mathcal{G}, B] \subseteq [\mathcal{H}, C]$ , then the following results hold

- (i)  $D_p([\mathcal{F}, A], [\mathcal{H}, C]) = D_p([\mathcal{F}, A], [\mathcal{G}, B]) + D_p([\mathcal{G}, B], [\mathcal{H}, C])$
- (ii)  $D_m([\mathcal{F}, A], [\mathcal{H}, C]) = D_m([\mathcal{F}, A], [\mathcal{G}, B]) + D_m([\mathcal{G}, B], [\mathcal{H}, C])$
- (iii)  $ND_p([\mathcal{F}, A], [\mathcal{H}, C]) = ND_p([\mathcal{F}, A], [\mathcal{G}, B]) + ND_p([\mathcal{G}, B], [\mathcal{H}, C])$

$$(iv) \quad ND_m([\mathcal{F}, A], [\mathcal{H}, C]) = ND_m([\mathcal{F}, A], [\mathcal{G}, B]) + ND_m([\mathcal{G}, B], [\mathcal{H}, C])$$

**Example 3.12.** Consider the T2SS stated in example 2.7. Then,  
 $D_p([\mathcal{F}, A], [\mathcal{G}, B]) = 6$  units,  $D_m([\mathcal{F}, A], [\mathcal{G}, B]) = 0.08$  units,  $ND_p([\mathcal{F}, A], [\mathcal{G}, B]) = 7$  units and  $ND_m([\mathcal{F}, A], [\mathcal{G}, B]) = 0.093$  units.

## 4 Entropy measure

In this section we propose the definition of entropy and an entropy measure for T2SS.

**Definition 4.1.** A T2SS  $[\mathcal{F}, A]$  is said to be equivalent to another T2SS  $[\mathcal{G}, B]$  if there exists two bijective functions  $\psi : A \rightarrow B$  and  $\Gamma : E_A \rightarrow E_B$  such that  $\psi(\mathcal{F}_x^*) = \mathcal{G}_x^*$  and  $\Gamma(\mathcal{F}_x^{**}) = \mathcal{G}_x^{**}$ , for all  $x \in X$ , where  
 $\mathcal{F}_x^* = \{\alpha \in A : x \in F_\alpha(\beta) \text{ for some } \beta \in E_A^\alpha\}$ ,  $\mathcal{F}_x^{**} = \cup_{\alpha \in A} \{\beta \in E_A^\alpha : x \in F_\alpha(\beta)\}$ ,  
 $\mathcal{G}_x^* = \{\alpha \in B : x \in G_\alpha(\beta) \text{ for some } \beta \in E_B^\alpha\}$ ,  $\mathcal{G}_x^{**} = \cup_{\alpha \in B} \{\beta \in E_B^\alpha : x \in G_\alpha(\beta)\}$ .  
If a T2SS  $[\mathcal{G}, B]$  is equivalent to  $[\mathcal{F}, A]$ , it is written as,  $[\mathcal{G}, B] \simeq [\mathcal{F}, A]$ .

**Remark 4.2.** In particular, if for any two T2SS  $[\mathcal{F}, A], [\mathcal{G}, B]$ ,  $[\mathcal{F}, A] \simeq [\mathcal{G}, B]$  then  $|A| = |B|$ ,  $|E_A| = |E_B|$ ,  $|\mathcal{F}_x^*| = |\mathcal{G}_x^*|$  and  $|\mathcal{F}_x^{**}| = |\mathcal{G}_x^{**}|$ .

**Definition 4.3.** A T2SS  $[\mathcal{F}, A]$  is said to be deterministic if

- (i)  $A \cap E_A = \varphi$
- (ii) for distinct  $\alpha_1, \alpha_2 \in A$  (i.e.  $\alpha_1 \neq \alpha_2$ ),  $E_A^{\alpha_1} \cap E_A^{\alpha_2} = \varphi$  and  $\mathcal{F}(\alpha_1) \tilde{\cap} \mathcal{F}(\alpha_2) = \tilde{\varphi}$ , the null type-1 soft set.
- (iii) for distinct  $\beta_1, \beta_2 \in E_A^\alpha$  (i.e.  $\beta_1 \neq \beta_2$ ),  $F_\alpha(\beta_1) \cap F_\alpha(\beta_2) = \varphi$ .
- (iv)  $\cup_{\alpha \in A} \{\cup_{\beta \in E_A^\alpha} F_\alpha(\beta)\} = X$

**Example 4.4.** Consider a T2SS  $[\mathcal{F}, A]$ , over the crisp universe  $X = \{x_1, x_2, x_3, x_4, x_5\}$  and the sets of parameters  $A = \{\alpha_1, \alpha_2, \alpha_3\}$ ,  $E_A = \{\beta_1, \beta_2, \beta_3, \beta_4\}$  defined as,  
 $\mathcal{F}(\alpha_1) = \left\{ \frac{\beta_1}{\{x_1, x_2\}} \right\}; \mathcal{F}(\alpha_2) = \left\{ \frac{\beta_2}{\{x_4\}}, \frac{\beta_4}{\{x_5\}} \right\}; \mathcal{F}(\alpha_3) = \left\{ \frac{\beta_3}{\{x_3\}} \right\}$   
The T2SS  $[\mathcal{F}, A]$ , mentioned above is a deterministic T2SS.

**Remark 4.5.** The null and the absolute T2SS, respectively denoted by  $\tilde{\Phi}$  and  $\tilde{\mathbb{A}}$  are not deterministic T2SS.

**Definition 4.6.** A mapping  $E_m : S_2(X, E) \rightarrow \mathbb{R}^+$ , where  $S_2(X, E)$  denotes the set of all type-2 soft sets over the soft universe  $(X, E)$ , is said to be a measure of entropy of a T2SS if and only if it satisfies the following conditions:

- (e1)  $E_m(\tilde{\Phi}) = 1$ ,  $E_m(\tilde{\mathbb{A}}) = 1$
- (e2)  $E_m([\mathcal{F}, A]) \leq E_m([\mathcal{G}, B])$  if  $[\mathcal{F}, A] (\neq \tilde{\Phi}) \subseteq [\mathcal{G}, B]$ .
- (e3)  $E_m([\mathcal{F}, A]) = 0$  if  $[\mathcal{F}, A]$  is a deterministic T2SS.
- (e4)  $E_m([\mathcal{F}, A]) = E_m([\mathcal{G}, B])$  iff  $[\mathcal{F}, A] \simeq [\mathcal{G}, B]$ .

**Definition 4.7.** For a T2SS  $[\mathcal{F}, A]$ , define  $E_m : S_2(X, E) \rightarrow \mathbb{R}^+$  as  

$$E_m([\mathcal{F}, A]) = \begin{cases} 1 & \text{if } [\mathcal{F}, A] = \tilde{\mathbb{A}} \text{ or } [\mathcal{F}, A] = \tilde{\Phi} \\ 1 - \frac{2|X|}{\sum_{x \in X} \{|\mathcal{F}_x^*| + |\mathcal{F}_x^{**}|\}} & \text{otherwise} \end{cases}$$

**Theorem 4.8.**  $E_m([\mathcal{F}, A])$  as defined above, is a measure of entropy for T2SS.

**Example 4.9.** For example 2.7, the entropy measure  $E_m([\mathcal{F}, A]) = 0.44$ .

## 5 Similarity measures

**Definition 5.2.** For two T2SS  $[\mathcal{F}, A], [\mathcal{G}, B]$ , define the soft real number valued similarity measure as a soft real valued mapping  $S' : (A \cup B) \rightarrow \mathcal{P}(R)$ , over the set of parameters  $A \cup B$ , such that for any two T2SS  $[\mathcal{F}, A], [\mathcal{G}, B] \in S_2(X, E)$  that are not disjoint, for each  $\alpha \in A \cup B$ ,  $S'(\alpha)$  is defined as  $S'(\alpha) = S_\alpha((F_\alpha, E_A^\alpha), (G_\alpha, E_B^\alpha))$  where  $S_\alpha((F_\alpha, E_A^\alpha), (G_\alpha, E_B^\alpha))$  is the similarity measure between the T1SS corresponding to each primary parameter  $\alpha$  i.e. it satisfies the conditions:

- (s'1)  $S_\alpha((F_\alpha, E_A^\alpha), (G_\alpha, E_B^\alpha)) = S_\alpha((G_\alpha, E_B^\alpha), (F_\alpha, E_A^\alpha))$
- (s'2)  $0 \leq S_\alpha((F_\alpha, E_A^\alpha), (G_\alpha, E_B^\alpha)) \leq 1$
- (s'3)  $S_\alpha((F_\alpha, E_A^\alpha), (G_\alpha, E_B^\alpha)) = 1$  iff  $(F_\alpha, E_A^\alpha) = (G_\alpha, E_B^\alpha)$
- (s'4) for  $(F_\alpha, E_A^\alpha), (G_\alpha, E_B^\alpha), (H_\alpha, E_C^\alpha) \in S(X, E)$ , where  $S(X, E)$  denotes the collection of all T1SS over the initial soft universe  $(X, E)$ , if  $(F_\alpha, E_A^\alpha) \subseteq (G_\alpha, E_B^\alpha) \subseteq (H_\alpha, E_C^\alpha)$  then  $S_\alpha((F_\alpha, E_A^\alpha), (H_\alpha, E_C^\alpha)) \leq S_\alpha((F_\alpha, E_A^\alpha), (G_\alpha, E_B^\alpha)) \wedge S_\alpha((G_\alpha, E_B^\alpha), (H_\alpha, E_C^\alpha))$
- (s'5)  $S_\alpha((F_\alpha, E_A^\alpha), (G_\alpha, E_B^\alpha)) = 0$ , for all  $\alpha \in A \cup B$  if  $[\mathcal{F}, A], [\mathcal{G}, B] \in S_2(X, E)$  are disjoint.

**Definition 5.3.** Suppose  $[\mathcal{F}, A], [\mathcal{G}, B] \in S_2(X, E)$ . Define for  $\alpha \in A \cup B$

$$S(\alpha) = \begin{cases} \frac{1}{|E_A^\alpha \cup E_B^\alpha|} \times \left\{ \sum_{\beta \in E_A^\alpha \cap E_B^\alpha} \frac{|F_\alpha(\beta) \cap G_\alpha(\beta)|}{|F_\alpha(\beta) \cup G_\alpha(\beta)|} \right\} & \text{when } \alpha \in A \cap B \neq \phi, E_A^\alpha \cap E_B^\alpha \neq \varphi \\ 0 & \text{otherwise} \end{cases}$$

**Theorem 5.4.**  $S$  is a soft real number valued similarity measure between  $[\mathcal{F}, A]$  and  $[\mathcal{G}, B]$ .

**Definition 5.5.** A mapping  $S_m : S_2(X, E) \times S_2(X, E) \rightarrow \mathbb{R}^+$ , where  $\mathbb{R}^+$  denotes the set of all positive reals and  $S_2(X, E)$  denotes the set of all type-2 soft sets over the soft universe  $(X, E)$ , is said to be a measure of similarity between any two T2SS if and only if for all  $[\mathcal{F}, A], [\mathcal{G}, B], [\mathcal{H}, C] \in S_2(X, E)$  it satisfies the following conditions:

- (s1)  $S_m([\mathcal{F}, A], [\mathcal{G}, B]) = S_m([\mathcal{G}, B], [\mathcal{F}, A])$
- (s2)  $0 \leq S_m([\mathcal{F}, A], [\mathcal{G}, B]) \leq 1$
- (s3)  $S_m([\mathcal{F}, A], [\mathcal{G}, B]) = 1$  iff  $[\mathcal{F}, A] = [\mathcal{G}, B]$ .
- (s4) if  $[\mathcal{F}, A] \subseteq [\mathcal{G}, B] \subseteq [\mathcal{H}, C]$  then  $S_m([\mathcal{F}, A], [\mathcal{H}, C]) \leq S_m([\mathcal{F}, A], [\mathcal{G}, B]) \wedge S_m([\mathcal{G}, B], [\mathcal{H}, C])$

**Theorem 5.6.** The mapping  $S_m : S_2(X, E) \times S_2(X, E) \rightarrow \mathbb{R}^+$  defined as

$$S_m([\mathcal{F}, A], [\mathcal{G}, B]) = \begin{cases} \frac{1}{|A \cup B|} \times \{ \sum_{\alpha \in A \cap B} S(\alpha) \} & \text{when } \alpha \in A \cap B \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

is a real valued similarity measure between the T2SS  $[\mathcal{F}, A], [\mathcal{G}, B] \in S_2(X, E)$ , where  $S(\alpha)$  is a soft real valued similarity measure between  $[\mathcal{F}, A], [\mathcal{G}, B]$ .

Proof: We give the proof of (s4). Other proofs can be done similarly.

- (s4) Suppose, for  $[\mathcal{F}, A], [\mathcal{G}, B], [\mathcal{H}, C] \in S_2(X, E)$ , such that,  $[\mathcal{F}, A] \subseteq [\mathcal{G}, B] \subseteq$



$[\mathcal{H}, C]$ . Thus, from the conditions of containment for T2SS,  
 $A \subseteq B \subseteq C$ ,  $\mathcal{F}(\alpha) \tilde{\subset} \mathcal{G}(\alpha)$ , for each  $\alpha \in A$  and  $\mathcal{G}(\beta) \tilde{\subset} \mathcal{H}(\beta)$ , for each  $\beta \in B$ .

Consider,  $S_m([\mathcal{F}, A], [\mathcal{H}, C])$  and  $S_m([\mathcal{F}, A], [\mathcal{G}, B])$ .

$\frac{|A \cap C|}{|A \cup C|} = \frac{|A|}{|C|}$  and  $\frac{|A \cap B|}{|A \cup B|} = \frac{|A|}{|B|}$ . Since,  $B \subseteq C$ ,  $|B| \leq |C|$ . Thus,

$$\frac{|A|}{|C|} \leq \frac{|A|}{|B|} \text{ i.e. } \frac{|A \cap C|}{|A \cup C|} \leq \frac{|A \cap B|}{|A \cup B|}.$$

Again, since  $\mathcal{F}(\alpha) \tilde{\subset} \mathcal{G}(\alpha) \tilde{\subset} \mathcal{H}(\alpha)$ , for each  $\alpha \in A$ ,  $E_A^\alpha \subseteq E_B^\alpha \subseteq E_C^\alpha$ . Hence,

$$\frac{|E_A^\alpha \cap E_C^\alpha|}{|E_A^\alpha \cup E_C^\alpha|} = \frac{|E_A^\alpha|}{|E_C^\alpha|} \leq \frac{|E_A^\alpha \cap E_B^\alpha|}{|E_A^\alpha \cup E_B^\alpha|} = \frac{|E_A^\alpha|}{|E_B^\alpha|}.$$

Also,  $|E_A^\alpha \cap E_C^\alpha| = |E_A^\alpha| = |E_A^\alpha \cap E_B^\alpha| \Rightarrow \sum_{\alpha \in A \cap C} |E_A^\alpha \cap E_C^\alpha| = \sum_{\alpha \in A \cap B} |E_A^\alpha \cap E_B^\alpha|$

Now,  $F_\alpha(\beta) \subseteq G_\alpha(\beta) \subseteq H_\alpha(\beta)$ ,  $\alpha \in A \cap B \cap C$ . Thus,  $\frac{|F_\alpha(\beta) \cap H_\alpha(\beta)|}{|F_\alpha(\beta) \cup H_\alpha(\beta)|} = \frac{|F_\alpha(\beta)|}{|H_\alpha(\beta)|} \leq$

$\frac{|F_\alpha(\beta)|}{|G_\alpha(\beta)|} = \frac{|F_\alpha(\beta) \cap G_\alpha(\beta)|}{|F_\alpha(\beta) \cup G_\alpha(\beta)|}$ . Hence,

$$\begin{aligned} & \frac{|A \cap C|}{|A \cup C|} \times \frac{\sum_{\alpha \in A \cap C} \left\{ \frac{|E_A^\alpha \cap E_C^\alpha|}{|E_A^\alpha \cup E_C^\alpha|} \times \frac{\sum_{\beta \in E_A \cap E_C} \frac{|F_\alpha(\beta) \cap H_\alpha(\beta)|}{|F_\alpha(\beta) \cup H_\alpha(\beta)|}}{\sum_{\alpha \in A \cap C} |E_A^\alpha \cap E_C^\alpha|} \right\}}{\frac{|A \cap C|}{|A \cup C|}} \\ &= \frac{|A|}{|C|} \times \frac{\sum_{\alpha \in A} \left\{ \frac{|E_A^\alpha|}{|E_C^\alpha|} \times \frac{\sum_{\beta \in E_A} \frac{|F_\alpha(\beta)|}{|H_\alpha(\beta)|}}{\sum_{\alpha \in A} |E_A^\alpha|} \right\}}{\frac{|A|}{|C|}} \\ &\leq \frac{|A|}{|B|} \times \frac{\sum_{\alpha \in A} \left\{ \frac{|E_A^\alpha|}{|E_B^\alpha|} \times \frac{\sum_{\beta \in E_A} \frac{|F_\alpha(\beta)|}{|G_\alpha(\beta)|}}{\sum_{\alpha \in A} |E_A^\alpha|} \right\}}{\frac{|A|}{|B|}} \\ &= \frac{|A \cap B|}{|A \cup B|} \times \frac{\sum_{\alpha \in A \cap C} \left\{ \frac{|E_A^\alpha \cap E_C^\alpha|}{|E_A^\alpha \cup E_C^\alpha|} \times \frac{\sum_{\beta \in E_A \cap E_C} \frac{|F_\alpha(\beta) \cap G_\alpha(\beta)|}{|F_\alpha(\beta) \cup G_\alpha(\beta)|}}{\sum_{\alpha \in A \cap C} |E_A^\alpha \cap E_C^\alpha|} \right\}}{\frac{|A \cap B|}{|A \cup B|}} \\ &\Rightarrow S_m([\mathcal{F}, A], [\mathcal{H}, C]) \leq S_m([\mathcal{F}, A], [\mathcal{G}, B]). \end{aligned}$$

In an exactly analogous manner, it can be shown that  $S_m([\mathcal{F}, A], [\mathcal{H}, C]) \leq S_m([\mathcal{G}, B], [\mathcal{H}, C])$  which completes the proof.

## 5.1 Relation among similarity, distance and entropy measures for Type-2 soft sets

**Theorem 5.1.1.** If  $D([\mathcal{F}, A], [\mathcal{G}, B])$  denotes any of the afore-mentioned proposed distance measure between any two T2SS  $[\mathcal{F}, A]$  and  $[\mathcal{G}, B]$  then  $S_d([\mathcal{F}, A], [\mathcal{G}, B]) = \frac{1}{1+D([\mathcal{F}, A], [\mathcal{G}, B])}$  is a measure of similarity between the T2SS concerned.

**Remark 5.1.2.** The similarity measure  $S_d([\mathcal{F}, A], [\mathcal{G}, B])$  derived in terms of the distance between two T2SS is termed as a distance-based similarity measure for T2SS.

**Theorem 5.1.3.** If  $E_m([\mathcal{F}, A])$  denotes any measure of entropy for a T2SS  $[\mathcal{F}, A]$  then  $S_E([\mathcal{F}, A], [\mathcal{G}, B]) = 1 - |E_m([\mathcal{F}, A] \sqcup [\mathcal{G}, B]) - E_m([\mathcal{F}, A] \cap [\mathcal{G}, B])|$  is a measure of similarity between the T2SS concerned.

Only the proof of (s3) is shown.

(s3) Since  $0 \leq E_m([\mathcal{F}, A]) \leq 1$ , for any  $[\mathcal{F}, A] \in S_2(X, E)$ , it implies that  $0 \leq S_E([\mathcal{F}, A], [\mathcal{G}, B]) \leq 1$

Also,  $1 - |E_m([\mathcal{F}, A] \sqcup [\mathcal{G}, B]) - E_m([\mathcal{F}, A] \cap [\mathcal{G}, B])| = 0$  whenever  $|E_m([\mathcal{F}, A] \sqcup [\mathcal{G}, B]) - E_m([\mathcal{F}, A] \cap [\mathcal{G}, B])| = 1$ . We have, from Remark 4.2,

$$|A \cup B| = |A \cap B|, |(\mathcal{F} \sqcup \mathcal{G})_x^*| = |(\mathcal{F} \cap \mathcal{G})_x^*| \text{ and } |(\mathcal{F} \sqcup \mathcal{G})_x^{**}| = |(\mathcal{F} \cap \mathcal{G})_x^{**}|.$$

$$|A \cup B| = |A \cap B| \Rightarrow A = B \dots \dots \dots (1)$$

Again,

$$|(\mathcal{F} \sqcup \mathcal{G})_x^*| = \{ \alpha \in A : x \in F_\alpha(\beta_1) \cup G_\alpha(\beta_2), \beta_1, \beta_2 \in E_A^\alpha \cup E_B^\alpha \}$$

$$\begin{aligned}
&= |(\mathcal{F} \cap \mathcal{G})_x^*| = |\{\alpha \in A : x \in F_\alpha(\beta_1) \cap G_\alpha(\beta_2), \beta_1, \beta_2 \in E_A^\alpha \cap E_B^\alpha\}| \dots \dots \dots (a) \\
&|(\mathcal{F} \cup \mathcal{G})_x^{**}| = |\cup_{\alpha \in A} \{\beta \in E_A^\alpha : x \in F_\alpha(\beta_1) \cup G_\alpha(\beta_2), \beta_1, \beta_2 \in E_A^\alpha \cup E_B^\alpha\}| \\
&= |(\mathcal{F} \cap \mathcal{G})_x^{**}| = |\cup_{\alpha \in A} \{\beta \in E_A^\alpha : x \in F_\alpha(\beta_1) \cup G_\alpha(\beta_2), \beta_1, \beta_2 \in E_A^\alpha \cap E_B^\alpha\}| \dots \dots \dots (b) \\
&\text{From (b) we have, } |E_A^\alpha \cup E_B^\alpha| = |E_A^\alpha \cap E_B^\alpha| \Rightarrow E_A^\alpha = E_B^\alpha = E', \text{ say} \dots \dots \dots (2) \\
&\text{From (a),(b) we can conclude, } |\{x : x \in F_\alpha(\beta) \cup G_\alpha(\beta), \beta \in E'\}| = |x : x \in F_\alpha(\beta) \cap G_\alpha(\beta), \beta \in E'|, \text{ for each } \alpha \in A. \\
&\Rightarrow |F_\alpha(\beta) \cup G_\alpha(\beta)| = |F_\alpha(\beta) \cap G_\alpha(\beta)|, \text{ for each } \alpha \in A \text{ and } \beta \in E'. \\
&\Rightarrow F_\alpha(\beta) = G_\alpha(\beta), \text{ for each } \alpha \in A \text{ and } \beta \in E'. \\
&\text{Thus, } S_E([\mathcal{F}, A], [\mathcal{G}, B]) = 1 \Leftrightarrow [\mathcal{F}, A] = [\mathcal{G}, B].
\end{aligned}$$

**Example 5.1.4** Consider the T2SS stated in example 2.7. Then,  $S_m([\mathcal{F}, A], [\mathcal{G}, B]) = 0.167$ . Also, the distance based similarity measures corresponding to the distance measure  $D_m$  and normalized distance measure  $ND_m$  are 0.125 and 0.915 respectively (refer example 3.12.).

## 6 Application of the proposed soft real number valued similarity measure

As for application of soft real valued similarity measure, we have considered the case of a person suffering from diabetes. Thus, quite naturally the person would be advised to consume meals that are low in carbohydrates and rich in proteins and fibres. We assume that out of homely atmosphere, the person encounters different choices of food items on the platter out of which he has to choose the best suited food items which would not cause any harm to his present health status. Algorithm for selecting the best suited food items from the available choices involves the following steps:

**Step 1:** Suppose that we have  $n$  number of menus from  $n$  pantries under consideration such that the available food are categorized and listed in the form of  $n$  number of T2SS  $[\mathcal{F}_i, A_i], i = 1, 2, \dots, n$  and let the ideal preference of food items be listed as T2SS  $[\mathcal{F}, A]$ .

**Step 2:** Calculate the soft real number valued similarity measures  $S'([\mathcal{F}, A], [\mathcal{F}_i, A_i])$  for  $i = 1, 2, \dots, n$ .

**Step 3:** For a primary parameter  $\alpha \in A$ , compare the values of  $S'([\mathcal{F}, A], [\mathcal{F}_1, A_1])(\alpha)$ ,  $S'([\mathcal{F}, A], [\mathcal{F}_2, A_2])(\alpha)$ , ... and  $S'([\mathcal{F}, A], [\mathcal{F}_i, A_i])(\alpha)$  and select  $\max_i \{S'([\mathcal{F}, A], [\mathcal{F}_i, A_i])(\alpha)\}$ .

If for  $i = k \in [1, n]$ ,  $S'([\mathcal{F}, A], [\mathcal{F}_k, A_k])(\alpha) = \max_i \{S'([\mathcal{F}, A], [\mathcal{F}_i, A_i])(\alpha)\}$ , then the selected pantry is the  $k^{th}$  pantry. Proceed to Step 4.

**Step 4:** The selected food items are obtained as  $\mathcal{F}(\alpha) \cap \mathcal{F}_k(\alpha)$ .

**Step 5:** Repeat Step 3 for all  $\alpha \in A$ .

**Step 6:** Stop.

We consider a set of two different menus from two different pantries and we proceed with representing the available food items in the respective menus in terms of two different T2SS as follows, the primary classifying parameters being the types of food available for breakfast, lunch, dinner and supper.

Suppose,  $X$  denote the set of all available food items given by  $X = \{\text{pastry, bagels, brown bread, mousse, noodles, rice, fruit juice, cereals, pasta,}$

*vegetables, club sandwich, chicken, salad, soup, fish, pudding, milk, fruits, egg, chapati, nuts*).

Let, the entire set of underlying parameters is given by,  $\{ \text{fibre rich, protein rich, carb. rich, soft diet, liquid diet} \}$  and the primary set of parameters be  $A = \{ \text{breakfast, lunch, dinner, supper} \}$ .

Suppose the food available at Pantry1 can be classified into a T2SS  $[\mathcal{F}_1, A]$  as follows:

$$\begin{aligned}\mathcal{F}_1(\text{breakfast}) &= \left\{ \frac{\text{carb. rich}}{\{\text{pastry, bagels}\}}, \frac{\text{fluid diet}}{\{\text{fruit juice}\}}, \frac{\text{fibre rich}}{\{\text{cereals, fruits}\}} \right\} \\ \mathcal{F}_1(\text{lunch}) &= \left\{ \frac{\text{carb. rich}}{\{\text{rice, noodles, pasta}\}}, \frac{\text{protein rich}}{\{\text{fish}\}}, \frac{\text{fibre rich}}{\{\text{vegetables, salad}\}} \right\} \\ \mathcal{F}_1(\text{dinner}) &= \left\{ \frac{\text{protein rich}}{\{\text{chicken}\}}, \frac{\text{soft diet}}{\{\text{soup}\}}, \frac{\text{fibre rich}}{\{\text{salad}\}} \right\} \\ \mathcal{F}_1(\text{supper}) &= \left\{ \frac{\text{carb. rich}}{\{\text{club sandwich}\}}, \frac{\text{soft diet}}{\{\text{pudding}\}} \right\}\end{aligned}$$

In a similar way, the food items available in Pantry2 is classified into a T2SS  $[\mathcal{F}_2, A]$  as follows:

$$\begin{aligned}\mathcal{F}_2(\text{breakfast}) &= \left\{ \frac{\text{carb. rich}}{\{\text{bagels}\}}, \frac{\text{protein rich}}{\{\text{egg, chicken}\}}, \frac{\text{fluid diet}}{\{\text{fruit juice, milk}\}}, \frac{\text{fibre rich}}{\{\text{brown bread, cereals, fruits}\}} \right\} \\ \mathcal{F}_2(\text{lunch}) &= \left\{ \frac{\text{carb. rich}}{\{\text{rice, noodles}\}}, \frac{\text{fibre rich}}{\{\text{vegetables}\}}, \frac{\text{soft diet}}{\{\text{mousse}\}} \right\} \\ \mathcal{F}_2(\text{dinner}) &= \left\{ \frac{\text{carb. rich}}{\{\text{noodles}\}}, \frac{\text{protein rich}}{\{\text{fish}\}}, \frac{\text{fibre rich}}{\{\text{chapati, vegetables}\}} \right\} \\ \mathcal{F}_2(\text{supper}) &= \left\{ \frac{\text{protein rich}}{\{\text{chicken}\}}, \frac{\text{fibre rich}}{\{\text{nuts, salad}\}} \right\}\end{aligned}$$

Finally we represent the ideal food items that the person is allowed to eat throughout the day for the various course of meals in terms of a T2SS  $[\mathcal{F}, A]$  as,

$$\begin{aligned}\mathcal{F}(\text{breakfast}) &= \left\{ \frac{\text{fibre rich}}{\{\text{cereals, fruits, brown bread}\}}, \frac{\text{fluid diet}}{\{\text{milk}\}} \right\} \\ \mathcal{F}(\text{lunch}) &= \left\{ \frac{\text{protein rich}}{\{\text{fish, chicken, egg}\}}, \frac{\text{fibre rich}}{\{\text{vegetables, salad, chapati}\}} \right\} \\ \mathcal{F}(\text{dinner}) &= \left\{ \frac{\text{protein rich}}{\{\text{chicken, fish}\}}, \frac{\text{soft diet}}{\{\text{soup}\}}, \frac{\text{fibre rich}}{\{\text{salad, vegetables, chapati}\}} \right\} \\ \mathcal{F}(\text{supper}) &= \left\{ \frac{\text{soft diet}}{\{\text{soup}\}}, \frac{\text{fibre rich}}{\{\text{salad}\}} \right\}\end{aligned}$$

We now attempt to solve our problem at hand.

Here,  $i = 2$  and  $A_1 = A_2 = A$ .

The soft real valued similarity measures are given as,

$$S'([\mathcal{F}, A], [\mathcal{F}_1, A]) = \{(\text{breakfast}, 0.444), (\text{lunch}, 0.333), (\text{dinner}, 0.611), (\text{supper}, 0.000)\}$$

$$S'([\mathcal{F}, A], [\mathcal{F}_2, A]) = \{(\text{breakfast}, 0.375), (\text{lunch}, 0.083), (\text{dinner}, 0.292), (\text{supper}, 0.167)\}$$

Thus, comparing the values parameterwise we see that for the primary parameter *breakfast*,  $\max_{i=1,2} \{S'([\mathcal{F}, A], [\mathcal{F}_i, A_i])\} = S'([\mathcal{F}, A], [\mathcal{F}_2, A])$ . We thus conclude that the person would opt for his breakfast from Pantry2. Also, from further calculations we see that the person opts for his *lunch* from Pantry1, *dinner* from Pantry1 and *supper* from Pantry2.

Also, for the choice of food items we show the calculations pertaining to the parameter viz. *breakfast* as,

$$\mathcal{F}(\text{breakfast}) \tilde{\cap} \mathcal{F}_2(\text{breakfast}) = \left\{ \frac{\text{fibre rich}}{\{\text{cereals, fruits, brown bread}\}}, \frac{\text{fluid diet}}{\{\text{milk}\}} \right\}$$

Thus, the choice of food items for breakfast include *cereals, fruits, brown bread, milk*. Similarly, for *lunch, dinner* and *supper*, the list of foods is given by,

$$\mathcal{F}(\text{lunch}) \tilde{\cap} \mathcal{F}_1(\text{lunch}) = \left\{ \frac{\text{protein rich}}{\text{fish}}, \frac{\text{fibre rich}}{\text{vegetables, salad}} \right\}$$

$$\mathcal{F}(\text{dinner}) \tilde{\cap} \mathcal{F}_1(\text{dinner}) = \left\{ \frac{\text{protein rich}}{\text{chicken}}, \frac{\text{soft diet}}{\text{soup}}, \frac{\text{fibre rich}}{\text{salad}} \right\}$$

$$\mathcal{F}(\text{supper}) \tilde{\cap} \mathcal{F}_2(\text{supper}) = \left\{ \frac{\text{fibre rich}}{\text{salad}} \right\}$$

i.e. for lunch, the permissible food items include *fish, vegetables, salad*; the preferable food for dinner include *chicken, soup, salad* and *salad* for supper.

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